C4 INTEGRATION — Worksheet K

1 Find the general solution of each differential equation.

a $\frac{dy}{dx} = (x + 2)^3$ **b** $\frac{dy}{dx} = 4\cos 2x$ **c** $\frac{dx}{dt} = 3e^{2t} + 2$

d $(2 - x)\frac{dy}{dx} = 1$ **e** $\frac{dN}{dt} = t\sqrt{t^2 + 1}$ **f** $\frac{dy}{dx} = xe^x$

2 Find the particular solution of each differential equation.

a $\frac{dy}{dx} = e^{-x}$, $y = 3$ when $x = 0$ **b** $\frac{dy}{dt} = \tan^3 t \sec^2 t$, $y = 1$ when $t = \frac{\pi}{3}$

c $(x^2 - 3)\frac{du}{dx} = 4x$, $u = 5$ when $x = 2$ **d** $\frac{dy}{dx} = 3\cos^2 x$, $y = \pi$ when $x = \frac{\pi}{2}$

3 **a** Express $\frac{x-8}{x^2 - x - 6}$ in partial fractions.

b Given that

$$(x^2 - x - 6)\frac{dy}{dx} = x - 8,$$

and that $y = \ln 9$ when $x = 1$, show that when $x = 2$, the value of y is $\ln 32$.

4 Find the general solution of each differential equation.

a $\frac{dy}{dx} = 2y + 3$ **b** $\frac{dy}{dx} = \sin^2 2y$ **c** $\frac{dy}{dx} = xy$

d $(x + 1)\frac{dy}{dx} = y$ **e** $\frac{dy}{dx} = \frac{x^2 - 2}{y}$ **f** $\frac{dy}{dx} = 2\cos x \cos^2 y$

g $\sqrt{x}\frac{dy}{dx} = e^{y-3}$ **h** $y\frac{dy}{dx} = xy^2 + 3x$ **i** $\frac{dy}{dx} = xy\sin x$

j $\frac{dy}{dx} = e^{2x-y}$ **k** $(y - 3)\frac{dy}{dx} = xy(y - 1)$ **l** $\frac{dy}{dx} = y^2 \ln x$

5 Find the particular solution of each differential equation.

a $\frac{dy}{dx} = \frac{x}{2y}$, $y = 3$ when $x = 4$ **b** $\frac{dy}{dx} = (y + 1)^3$, $y = 0$ when $x = 2$

c $(\tan^2 x)\frac{dy}{dx} = y$, $y = 1$ when $x = \frac{\pi}{2}$ **d** $\frac{dy}{dx} = \frac{y+2}{x-1}$, $y = 6$ when $x = 3$

e $\frac{dy}{dx} = x^2 \tan y$, $y = \frac{\pi}{6}$ when $x = 0$ **f** $\frac{dy}{dx} = \sqrt{\frac{y}{x+3}}$, $y = 16$ when $x = 1$

g $e^x\frac{dy}{dx} = x\operatorname{cosec} y$, $y = \pi$ when $x = -1$ **h** $\frac{dy}{dx} = \frac{1+\cos y}{2x^2 \sin y}$, $y = \frac{\pi}{3}$ when $x = 1$